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The project concerns the development of effective numerical methods for the solution of stochastic control problems. In order to focus on fundamental issues, the bulk of the model applications are taken from current problems in high speed telecommunications, but the results are applicable in stochastic control in general.

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NUMERICAL METHODS IN STOCHASTIC CONTROL

FINAL PROGRESS REPORT

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AUGUST 19, 1996

U.S. ARMY RESEARCH OFFICE

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Numerical Methods in Stochastic Control

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The problem studied

The project concerns the development of effective numerical methods for the solution of stochastic control problems. In order to focus on fundamental issues, the bulk of the model applications are taken from current problems in high speed telecommunications, but the results are applicable in stochastic control in general.

Results

Two categories of problems are of interest. The first concerns large deviations methods for controlled queueing networks of the type that arise in communication. Desired error rates (i.e., noise effects) are generally exceedingly small, cannot be estimated directly, and require the methods of large deviation theory. Numerical methods are essential to get good estimates on performance, and to enable good designs. Our numerical methods are the only ones available at this time. The second category of problems is the optimization of applications which can be approximated by controlled diffusions, or diffusion or jump diffusion processes, as is the case for a large number of the telecommunications problems of interest. This approximation is possible for a large number of important problems in the subject areas. Our Markov chain approximation method is the current method of choice. The original problem is approximated by a computable problem which is still physically close to it. Efficient algorithms codes have been developed and tested on practical problems with excellent results. More detail follows.

The first set of problems studied involves the analysis of and development of numerical methods for communication and computer networks, where the noise effects are small. It is frequently necessary that the "error" rates of the system be quite small. An "error" might be a blocked transmission or loss of a data packet (say, with probability of order 10^{-8} or smaller). Direct calculation of such probabilities is memory intensive and slow, if not impossible. In addition, such calculations are not very informative, since they provide only a mass of numbers with little information on how different parts of the network interact. The goal of the analysis is often to understand how errors can be avoided, and for design or regulation purposes one needs more information. Direct simulation in such cases is also usually out of the question, and in any case it would not give the qualitative information needed for design. Since experience dictates that the probabilities will scale in an exponential fashion with respect to a parameter that measures the "capacity" of the system, e.g, the buffer sizes, an

asymptotic (large deviations) approach is natural. These techniques are naturally suited to the difficult types of problems where rare events are important, and conventional intuition not very useful. The large deviations approximations are defined in terms of the solution to a variational problem, and it is the minimizing parameter in this variational problem that identifies the most likely way in which the undesirable event can occur. However, the discontinuities and other nonstandard features of communication networks make the large deviation analysis of such systems very difficult, and relatively few results are available for networks. These quatities are hard to compute and numerical methods are essential. Their development is based an an alternative and powerful appoach to the general theory of large deviations, which also faciltates the analysis of the networks themselves.

A key ingredient in our approach to large deviations are representations for the normalized quantities of interest. Since large deviation expectations and probabilities scale in an exponential fashion, it is natural to phrase the convergence statement in terms of the normalized logarithm of the quantity of interest. The representations facilitate this analysis significantly, and allow one to substitute the well developed methods of weak convergence theory for the relatively awkward exponential estimates used in standard approaches. The representation is as the solution to a variational problem, and in all cases it takes the form of a small noise stochastic control problem. Besides their use in the large deviation analysis, the representations are also important because they provide the starting point for approximations. Furthermore, the methods used for the large deviations convergence can frequently be applied to prove convergence of these approximations.

The paper [A] and the book [B] develop representations for a wide variety of stochastic systems. [B] focuses largely on discrete time processes in a general framework, while [A] is devoted to continuous time Markov models for queueing networks. Besides setting up the representations, [A] proves their convergence under weak conditions and provides a means for identifying the limit problem variational problem.

The paper [C] develops the theory for networks in a different direction. Instead of laying the foundations for a very general theory, this paper concentrates on a particular non-Markovian network model that is currently of interest in the context of high-speed data networks. It is the first and only paper to date that allows for a complete large deviations analysis of bursty data sources in a network environment. In particular, in [C] the variational problem that characterizes the large deviation approximation is solved explicitly, thus providing a concrete characterization of how the interactions between different parts of the network lead to significant data loss. The methods can be generalized to other network and data models.

Lastly there is the paper [D], which is concerned with rates of convergence for approximations in stochastic and deterministic control. The paper presents a simple method for obtaining rate of convergence which is applicable to a wide

range of approximation problems. Heavy use is made of representations of the approximations as functionals of a controlled Markov chain, which of course is a by-product of the Markov chain approximation method. The method is illustrated thought the presentation of a number of examples, including finite difference schemes for stochastic and deterministic optimal control problems with several different cost structures. A general principle can be abstracted, and indeed the method may be applied to a variety of approximation problems, such as the numerical approximation of nonlinear PDE not a priori related to control theory.

Concerning the second class of problems, codes have been written, and applied very successfully to problems in telecommunications, yielding important information on the performance and design which is not available by other means.

We have developed domain decomposition methods for the numerical solution of Markov chain control problems on large state spaces of the type which arise as approximations to optimal stochastic control problems with diffusion, jump-diffusion or reflected diffusion type models. These are also applicable to the degenerate nonlinear elliptic partial differential integral equations or variational inequalities of the type which appear as "Bellman" equations. Whether or not the PDE has only a formal meaning, the solutions to the approximating Markov chain problems converge to the desired cost or optimal functional. Owing to many non standard features in such problems and to the nonlinearity of the equations which are to be solved, the decomposition problems for the Markov chain control problems often require special methods. We prove that appropriate adaptations of current decomposition techniques converge under conditions which are typical of many classes of applications. Probabilistic interpretations of the various algorithms in terms of functionals of Markov chains (controlled or not) are used. An important application is to problems in four or more dimensions where, without decomposition, the size and structure of the state space and associated data structures often leads to very poor performance on cache machines.

Papers and reports

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Software available on the internet

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